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# Four Dimensions and Reference "Frames": Some Operational Interpretations

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*Abstract*

Operational interpretations of the four-dimensionality of our universe and of the reference "frames" of relativity theory are presented. The interpretations display a property intriguingly similar to the Pauli exclusion principle of quantum physics.

In this paper, operational interpretations of the four dimensions of our universe and of the reference "frames" of relativity theory are given. These interpretations involve a "geometric Pauli property" intriguingly similar to the Pauli exclusion principle of quantum physics.

We begin with an operational interpretation of the familiar "event" of relativity theory (Taylor and Wheeler, 1966). The event is operationally identifiable. That is, several observers can and do agree that a particular event has occurred and that it is one and the same event. For example, consider the flash of a flashbulb. Everyone who observes the flash can agree that there was an event and that there was just one event. No nonobserver denies that there was an event. And furthermore all observers and nonobservers can and do agree, in some approximation, that only one flashbulb flashed. We know from experience that we can and do agree on such things. Other examples of operationally identifiable events are collisions. For example, collisions between two trains or two cars, the touchdown of an aircraft, the catching of a ball, the kicking of a ball, are all operationally identifiable events. We then view the universe as the set of all such operationally identifiable events.

The four dimensions of the universe then might have the following operational interpretation: the events which constitute the nearby universe are so connected that we can assign a unique ordered 4-tuple of real numbers to each constituent event. "Unique" here means that no two events share the same ordered 4-tuple. Furthermore, the connection is such that the ordered

4-tuples can be assigned so that they are "well behaved," that is, are continuous and differentiable.

Clarification of the interpretation of four dimensions just given requires operational definitions for "nearby universe," "continuity," and "differentiability." In order to make these definitions, we shall first develop some other concepts operationally. In particular, we shall consider an operational meaning for reference "frames." First, consider an ordered 4-tuple assignment on some set of events, $S$. In general, there are many distinctly different such assignments that can be made on $S$. We consider one in particular, say the $H$th assignment. One way to specify the $H$th assignment is to list in one column of a table all the events in the set under consideration. We would represent each event in this list with a description of operations which would uniquely identify it. In the other column of the table, opposite the operational description of the event, we would list the ordered 4-tuple assigned to it by assignment $H$. If the assignment $H$ is continuous, differentiable, and such that no two events in $S$ have the same 4-tuple assigned, then we shall call assignment $H$ a reference assignment, or coordinate assignment, or reference system, or coordinate system, for the set $S$. Such a continuous, differentiable reference assignment is, indeed, the essence of the "coordinate systems" or "reference frames" of standard works in geometry and physics (Taylor and Wheeler, 1966). Given an arbitrary event $I$ in the set $S$, we shall represent the ordered 4-tuple assigned to $I$ by assignment $H$ with the notation $({}^Ht_I, {}^Hx_I, {}^Hy_I, {}^Hz_I)$.

We turn now to the "nearby universe." The meaning of "nearby universe" involves certain types of subsets $S$. In particular, a subset $S$ of the universe, where subset $S$ includes event $E$, is said to be the "universe nearby $E$" if and only if (a) there exists a real number $\delta > 0$ such that every event within radar echo range $\delta$ of event $E$ is in set $S$, (b) the set $S$ includes at least one event distinct from $E$, and (c) the set $S$ is "four-dimensional," i.e., its geometry has the same set of properties as does the geometry of any finite subset of the universe. "Finite" means every event in the set is within radar echo range $\gamma$ of every other event in the set, with $\gamma$ finite. By "within radar echo range $\delta$ or $\gamma$ of event $E$" is meant the following: Consider the world line of some operationally realizable clock $C$ which experiences, i.e., "goes through," event $E$ (footnote 1). Let the number ${}^Ht_E$ assigned by assignment $H$ to event $E$ be just the time clock $C$ registers at event $E$. That is, ${}^Ht_E$ is the "proper time" of clock $C$ at $E$ (Taylor and Wheeler, 1966). Furthermore, we shall take assignment $H$ to be such that it assigns to any event $D$ experienced by clock $C$ just the time registered by $C$ at that event $D$. This "$H$-time," the time assigned by $H$ to $D$, is represented by ${}^Ht_D$. It is assigned by $H$ to be just the "proper time" registered by clock $C$ at the event $D$. The set of events such as $D$ which clock $C$ experiences is called $C$'s "world line" (Taylor and Wheeler, 1966). Now, an event $I$ is "within radar echo range $\delta$ of $E$" if and only if a radar signal sent out by clock $C$ at a time on $C$ between ${}^Ht_E - \delta/2$ and ${}^Ht_E + \delta/2$ "bounces off $I$" and returns, i.e.,

[1] It is a remarkable property of our universe that such clocks exist, in principle, for every event $E$.

echoes back, to $C$, arriving back at $C$ between ${}^Ht_E - \delta/2$ and ${}^Ht_E + \delta/2$. A reminder here: the preceding superscript "$H$" on ${}^Ht_E$ or ${}^Ht_D$ indicates the $H$th assignment of ordered 4-tuples on some subset of the universe; i.e., it indicates reference system or reference assignment $H$, defined on the subset $S$ of the universe.

The "nearby universe" is the universe nearby the event "here and now."

We are now prepared to operationally define "continuity" and "differentiability," terms which apply to 4-tuple assignments defined on the universe nearby some event $E$. One candidate for an operational definition of continuity is the following: An ordered 4-tuple assignment $H$ is continuous at event $E$ if and only if, for every real number $\epsilon > 0$, there exists a real number $\delta > 0$ such that each number in the 4-tuple assigned by $H$ to any event $I$ within radar echo range $\delta$ of $E$ is within $\epsilon$ of the corresponding number in the ordered 4-tuple assigned by $H$ to $E$. There must be at least one such event $I$, operationally distinct from $E$, for every $\epsilon > 0$.

For example, suppose $E$ were assigned by assignment $H$ the ordered 4-tuple $({}^Ht_E, {}^Hx_E, {}^Hy_E, {}^Hz_E)$. Then, in order for a 4-tuple assignment on the universe to be continuous at event $E$, for every $\epsilon > 0$ there must be a $\delta > 0$ such that every event $I$ within radar echo range $\delta$ of $E$ has a 4-tuple assigned to it by $H$ between $({}^Ht_E - \epsilon, {}^Hx_E - \epsilon, {}^Hy_E - \epsilon, {}^Hz_E - \epsilon)$ and $({}^Ht_E + \epsilon, {}^Hx_E + \epsilon, {}^Hy_E + \epsilon, {}^Hz_E + \epsilon)$. There must be at least one such event $I$, other than $E$ itself, for every $\epsilon$ and $\delta$.

We can operationally define differentiability of the 4-tuple assignment $H$ at $E$ as follows: let $({}^Ht_E, \ldots, {}^Hz_E)$ represent the 4-tuple assigned by $H$ to event $E$, and let $({}^Ht_I, \ldots, {}^Hz_I)$ represent the 4-tuple assigned by $H$ to arbitrary event $I$. Then the 4-tuple assignment $H$ on the universe nearby $E$ is differentiable at $E$ if and only if the four limits

$$\lim_{\delta_I \to 0} \left[\frac{{}^Ht_E - {}^Ht_I}{\delta_I}\right]; \lim_{\delta_I \to 0} \left[\frac{{}^Hx_E - {}^Hx_I}{\delta_I}\right]; \lim_{\delta_I \to 0} \left[\frac{{}^Hy_E - {}^Hy_I}{\delta_I}\right]; \lim_{\delta_I \to 0} \left[\frac{{}^Hz_E - {}^Hz_I}{\delta_I}\right]$$

exist as $\delta_I$, the radar echo range to event $I$, goes to zero. Here we let the event $I$ vary in order to vary the radar echo range $\delta_I$. This radar echo is relative to some operationally realizable clock $C$ which experiences event $E$.

With these operational definitions, we have now an operational understanding of the interpretation of the four dimensions made above. We reiterate that interpretation: The operationally identifiable events which constitute the universe nearby us, here and now, are so connected that we can assign a unique ordered 4-tuple of real numbers to each constituent event, and this 4-tuple assignment can be made such that it is continuous and differentiable. Note that this remarkable property of the universe, its four-dimensionality, means that we can make a continuous 4-tuple assignment of real numbers such that no two events are assigned the same 4-tuple, i.e., the same four numbers. In other words, we need assign no two events the same set of four numbers. This statement sounds intriguingly like the Pauli exclusion principle of quantum mechanics: no two electrons can be assigned the same set of four quantum numbers. It is therefore suggested that this property of the geometry of the universe be called the "geometric Pauli property." One is led to guess that the similarity of these two "Pauli principles" reflects a fundamental connection between quantum physics and the four-dimensionality of the geometry of our experience. The author has suggested another such fundamental connection between quantum physics and geometry elsewhere (Nickerson, 1975).

## *References*

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